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Abstract. A method for effective focal length measurement using imaging conjugates is discussed and demonstrated. This method is used to determine the effective focal length of an objective lens with precision and without the need to know the exact position of the principal planes by measuring relative distances of imaging conjugates. Focal length determination was done with the aid of an interferometer and with a precision of ±0.054%. A discussion of the method is presented and an error analysis discussed. This method can be used for characterizing optical systems with a wide range of focal lengths because of its simple experimental configuration. [DOI: 10.1117/1.OE.51.11.113604]

Subject terms: focal length; effective focal length; focal length measurement; interferometer; nodal slide.

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1 Introduction

The effective focal length, or focal length, is a basic and important parameter of an optical system. Several methods for effective focal length measurement have been proposed and used in the literature for a long time. Although some of these methods are well known and achieve good precision,[1–7] a simple method with high precision is still desirable for many applications. By definition, the effective focal length (or rear focal length) is the distance measured from the second principal plane to the rear focal point. However, in some methods, such as the nodal slide method, not only is the nodal point difficult to find, but also the distance from the nodal point to the rear focal point is difficult to determine. These are drawbacks that complicate doing an error analysis and achieving high precision measurements. Some modern methods, such as Talbot interferometry,[8–10] moiré deflectometry,[11,12] and interferometery can provide high precision focal length measurements. However, often the experimental setup is complicated and this may induce more error sources. Other recent methods have been proposed using a grating shearing interferometer,[13] a Fresnel-zone hologram,[14] and digital Fourier transforms.[15] Table 1 summarizes some representative technologies for focal length measurement. The relationship of the measurement accuracy and the focal length range is also illustrated (Fig. 1). Although some of these methods provide high accuracy for specific focal length ranges, the measurement precision is not often discussed.

In this paper, we demonstrate a precise, simple method for measuring the effective focal length of a microscope objective. This method utilizes the relative distances of the conjugates imaged by the objective for determining the effective focal length. Without the need to know the position of the principle planes, the error sources are minimized and are easy to be analyzed.

2 Experimental Theory

To use imaging conjugates of an optical system for determining the effective focal length, three conjugates are required. An arrangement is shown in Fig. 2. One conjugate is selected at infinity and two others are selected at finite distances. Distances, O_1, O_2, O_3, I_1, I_2, and I_3 represent the object and image distances for the conjugates. H and H'' are the front and rear principal planes, respectively. The Gaussian imaging formula can be used to relate these parameters with the effective focal length f'':

$$\frac{1}{S''} = \frac{1}{S} + \frac{1}{f''}, \tag{1}$$

where S'' and S are the image and objective distances. To simplify we set the quantities $(I_2 - I_1) = A$, $(I_3 - I_1) = B$, and $(O_2 - O_3) = C$ which indicate the relative distances from the image point 1 to 2, the image point 1 to 3, and the object point 2 to 3, respectively. Then we can write the relationship between the focal length and the quantities A, B, and C,

Table 1 Technologies for focal length measurement.

Reference	Method	Features
8	Talbot interferometer	• Moiré effect utilized • Feasible for long FL
12	Moiré interferometer	• Moiré effect utilized • For any lens power
13	Grating shearing	• Only one grating used
15	Digital Fourier transform	• FFT for spatial freq. evaluation
14	Fresnel-zone hologram	• High precision for large, slow lenses

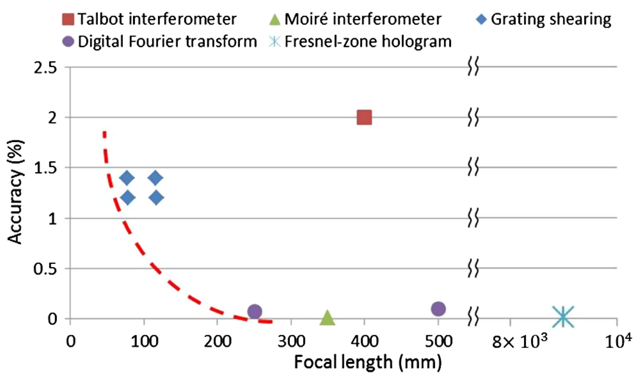

Fig. 1 Measurement accuracy of the methods.

$$\begin{cases} I_1 = f'' \\ \frac{1}{I_1+A} = \frac{1}{O_3+C} + \frac{1}{f''} \Rightarrow f'' = \pm\sqrt{\frac{ABC}{A-B}}. \\ \frac{1}{I_1+B} = \frac{1}{O_3} + \frac{1}{f''} \end{cases} \tag{2}$$

For a positive value for the focal length the positive root "+" is used in Eq. (2). This result shows that it is possible to determine the effective focal length of an optical system without knowing the positions of H and H''. Once the relative distances A, B, and C are measured, the effective focal length can be directly calculated from Eq. (2).

Another simple method can be used to determine the effective focal length through the following equation that uses only two conjugates,

$$f'' = \frac{L^2 - d^2}{4L}, \tag{3}$$

where L, d, and f'' are the distances between object and image, the separation of the two conjugates, and the effective focal length, respectively. However, in this approach determining the distance L can be a significant problem.

3 Error Analysis

For the error analysis a nominal collimated beam is assumed. Then only error sources from the measurement of the relative

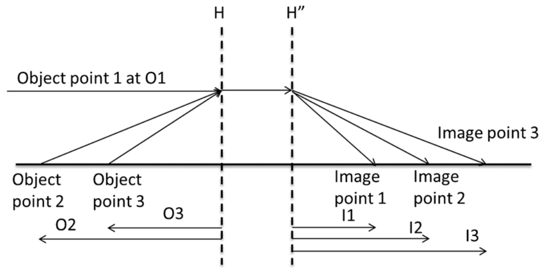

Fig. 2 The configuration of the experiment for focal length measurement.

distances A, B, and C are considered. Therefore, the sensitivity of effective focal length, f'', to the relative distances A, B, and C are found by taking partial derivatives of Eq. (2). By rearranging the differential results and setting $B = k \cdot A$ the error of the measurement can be expressed as follows,

$$E = \Sigma\partial f'' = \frac{1}{2}\left[-\frac{f''}{A}\frac{k}{(1-k)}\partial A + \frac{f''}{A}\frac{1}{k(1-k)}\partial B + \frac{A}{f''}\frac{k}{k(1-k)}\partial C\right], \tag{4}$$

where E is the sum of the error sources ∂A, ∂B, and ∂C. Equation (4) shows that the measurement error is affected not only by errors ∂A, ∂B, and ∂C but also by the selection of conjugates as shown in Fig. 2. Both the distance A and the k value selected can reduce the error in the results or enhance the measurement precision. In practical measurements the object distances are fixed, and the distances A and B are measured according to the spot size of the image point by a CCD sensor or by the fringe pattern in an interferometer. The error on the distance C can result from the measurement of the object position. In our experiment we concentrated on the measurement error of the conjugates positions when moving the object. Therefore, to simplify the analysis, the error of distance C was ignored. Hence, the error from $C(\partial C)$ was assumed to be zero. By expressing E as the standard deviation, Eq. (4) can be represented as,

$$\sigma_{f''} = \sqrt{\left|\frac{\partial f''}{\partial A}\right|^2 \sigma_A^2 + \left|\frac{\partial f''}{\partial B}\right|^2 \sigma_B^2}, \tag{5}$$

where σ_f, σ_A, and σ_B are the standard deviations of measurement of E, ∂A, and ∂B, respectively.

4 Measurement Setup

As Fig. 3 shows we arranged for a switchable setup between collimated and point light source illumination as this was needed for object 1 (at $c0$), and objects 2 and 3 (at $c1$ and $c2$). By switching the light source and producing a relative movement, the three conjugates of $c0$, $c1$, and $c2$ were easily set. The conjugates distances ($\varnothing$, a, and b) were determined by a CCD sensor placed in image space to observe the corresponding image points. Both the light source and CCD sensor were mounted on a high precision linear stage that could be moved along the optical axis. Once a best focus was detected, the corresponding position $\varnothing$ was recorded when the light source was in collimated mode. This step could be repeated for objects $c1$ and $c2$ with the point

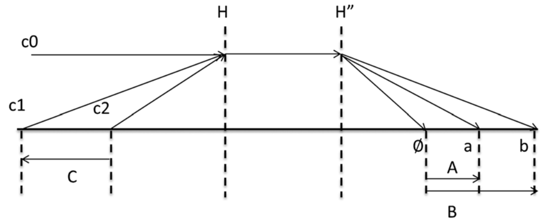

Fig. 3 The measurement configuration set-up.

light source mode and produced image points for positions a and b. One measurement was done and the relative distances A, B, and C were determined.

This basic setup was used to determine the A, B, and C values. One problem is that there may be bias errors due to spherical aberration since the measured objective is not corrected for the three conjugate distances used. This problem can be mitigated by using interferometry to determine focus rather than using focus spot sizes as shown by a CCD sensor.

4.1 *Measurement Using an Interferometer*

In order to eliminate the effects of spherical aberration we added an interferometer to our test set-up. Then we fixed the position of the illumination point source on one side of the objective lens and moved the objective to control the conjugate distances. A flat mirror was placed on the other side of the objective lens to reflect the light back to the lens and interferometer as shown in Fig. 4. This interferometer is a WYKO 6000 interferometer and produced a point source with a transmission sphere lens. In beginning the measurement, two positions for the mirror were selected to simulate the object positions $c1$ and $c2$. The case of infinity conjugate for object $c0$ was achieved by moving the objective lens until a no-focus fringe pattern was observed. In the analysis software of the interferometer, some small aberrations were present and were neglected. Therefore, the conjugate positions with zero defocus were determined by finding the distance where the “power” term, as shown in the analysis software, was zero. When the light beam was properly in focus, there was in the interferometer monitor a straight fringe pattern. To find A, B, and C for focal length calculation, these relative distances were measured by the position of the objective lens, and the mirror, according to each conjugate as Fig. 5 illustrates.

In this experiment a NIKON M PLAN 5X 0.1 microscope objective lens was used. The positions were determined when variation of the power values in the interferometer analysis

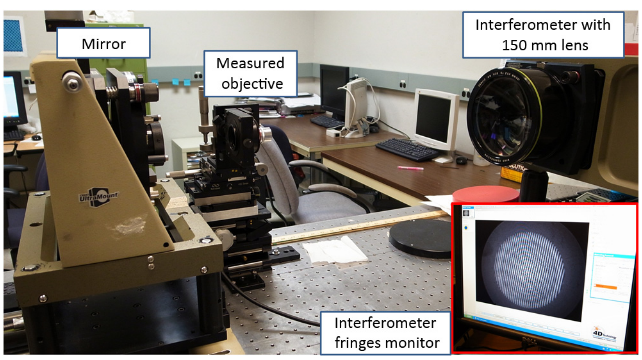

Fig. 4 The experimental arrangement with the interferometer.

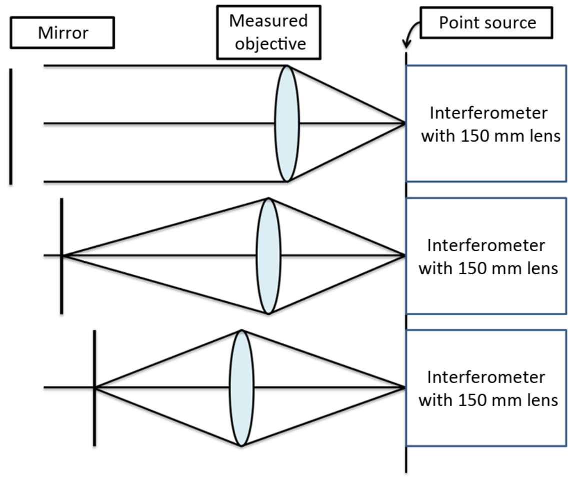

Fig. 5 The way to find the conjugates for focal length calculation using the interferometer and a mirror.

were within ± 5 nm wave front rms. By relatively moving the positions of objective and the mirror, the conjugate positions were determined and consequently the quantities A, B, and C. The measurement results are shown in Table 2.

Two positions were selected for $c1$ and $c2$, and then the positions $\varnothing$, a, and b were measured using the corresponding conjugates. The ratio of $B/A = k$ is approximately 4.56. This finally resulted in a $\pm 0.0538\%$ standard deviation error for the measured focal length. Compared to the theoretical derivation, the error in focal length E is $\pm 0.0521\%$ when the average values of A, B, f'', and k, and standard values, ∂A and ∂B, are substituted into Eqs. (4) and (5). These results show good measurement precision and agreement with the theoretical error analysis.

5 Non-Collimated Light Source Analysis

If the beam of light is not quite collimated, which means the object point 1 (O_1) is not at infinity, then there is a small shift in the image distance I_1,

Table 2 Measurement results by interferometry.

	$f'' = 37.4291 \pm 0.0201$ Error $= 0.0538\%$		
$k = 4.5592$ **C** $= -480$	A	B	f''
1	2.2759	10.3776	37.4074
2	2.2797	10.3808	37.4458
3	2.2803	10.3858	37.4496
4	2.2797	10.3909	37.4407
5	2.2767	10.4047	37.4021
AVG	2.2785	10.3880	37.4291
STD	0.0018	0.0095	0.0201
			Unit: mm

$$I_1 = f'' - \varepsilon. \tag{6}$$

We can obtain the following relationship by substitution of Eq. (6) in Eq. (1)

$$\varepsilon = \frac{f''^2}{f'' + O_1}. \tag{7}$$

Equation (7) shows that the error ε is increased when the beam is not collimated. This case of non-collimation results in a change of A and B, and then the focal length becomes,

$$f_e''(\varepsilon') = f'' \cdot \left(1 + \frac{1}{2}\varepsilon'\right), \tag{8}$$

where

$$\varepsilon' = \frac{(A+B)\varepsilon + \varepsilon^2}{AB} \tag{9}$$

and $f_e''(\varepsilon')$ is the focal length. The quantity ε' is expressed in terms of ε. f'' is the focal length without collimation error. Equations (8) and (9) can be derived when A and B in Eq. (2) are substituted with $A' = A + \varepsilon$ and $B' = B + \varepsilon$, which reflect that A and B change due to the tiny shift in the image distance I_1. For simplification we use a Taylor expansion so that $f_e''(\varepsilon')$ can be expressed as Eq. (8).

To relate ε to the degree of collimation, the object distance O_1 can be represented by the incident angle, and the NA of the measured microscope objective lens. Then Eq. (7) can be rewritten as,

$$\varepsilon = \frac{\tan\theta}{\tan\theta - \mathrm{NA}} \cdot f'', \tag{10}$$

where $\tan\theta = -D/O_1$, and the lens NA is approximately $D/2f''$. The parameter D is the lens aperture. By substitution of Eqs. (9) and (10) for Eq. (8) allow us to express the focal length with error as,

$$f_e'' = f'' + K_1 f''^2 + K_2 f''^3, \tag{11}$$

where K_1 and K_2 are

$$\begin{cases} K_1 = \dfrac{1}{2}\left(\dfrac{A+B}{AB}\right) \times \dfrac{\tan\theta}{\tan\theta - \mathrm{NA}} \\ K_2 = \dfrac{1}{2}\dfrac{1}{AB} \times \left(\dfrac{\tan\theta}{\tan\theta - \mathrm{NA}}\right)^2 \end{cases}. \tag{12}$$

For the case of the results in Table 2, assuming a 0.1 arc second collimation error, a lens with a NA = 0.1, and noting that the average A and B are 2.2785 and 10.3880, respectively, the error in focal length determination would be −0.005% compared to the system without collimation error.

6 Conclusion

We demonstrated a method for precise focal length determination that is based on imaging conjugates. The method uses relative distances of imaging conjugates to determine the effective focal length of a lens system. By not measuring absolute distances but relative distance measurements, we minimize error sources and hence improve the measurement precision. In our work, a microscope objective was tested and its focal length determined to 0.054% precision. We found agreement between the measurement errors and the theoretical error prediction. Furthermore, because of the advantage of the simple experimental configuration, this method can be utilized for measurements for a wide range of focal lengths and can be a candidate for high precision focal length measurement of optical systems.

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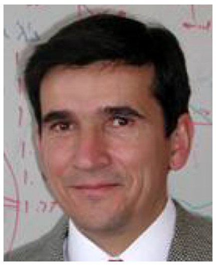

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Biogrpahy and photograph of the other author not available.